A Method of Forming a Broad-Band Microwave Frequency Spectrum

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Summary—A method of generating a wide spectrum of evenly spaced sidebands in the microwave region, suitable for use as a frequency standard, is presented and discussed. The frequency spectrum is produced by modulating the beam acceleration voltage of a klystron with two high-frequency voltages. The frequency of one of these modulation voltages is an integral multiple of the frequency of the other voltage, so that certain sidebands of the lower frequency coincide with the sidebands of the higher frequency. The output of the klystron then consists of a frequency spectrum about the microwave carrier frequency with the spacing between sidebands equal to the lowest of the two modulating frequencies.

The method is examined analytically and experimentally. The operation of a klystron is ordinarily expressed with time as the variable. An analysis making use of the Laplace transform to convert the function of time into a function of frequency has been found convenient. The result gives the magnitude of the sidebands as a function of the operating parameters of the klystron.

INTRODUCTION

THE PURPOSE of this paper is to present a method for generating a wide band of evenly spaced frequencies in the microwave region from a microwave carrier voltage and two high-frequency voltages. The frequency of one of the high-frequency voltages is to be an integral multiple of the frequency of the other high-frequency voltage. It is proposed that this wide band of accurately determinable frequencies is suitable for use in a microwave frequency standard.

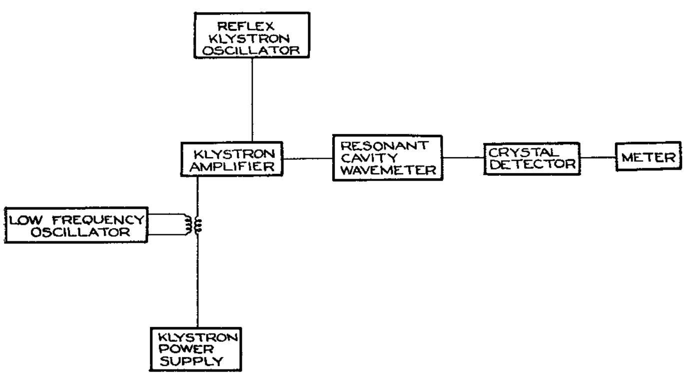

Fig. 1—Block diagram of laboratory set-up.

The method of generating this band of frequencies as it was done in this preliminary investigation is shown in Fig. 1. The proposed method for use in a frequency standard would differ somewhat from the configuration shown in Fig. 1. As an example of a typical design, a 5-megacycle crystal oscillator might be used, then a frequency multiplier to obtain 45-megacycle energy from the 5-megacycle oscillator; another ninth harmonic frequency multiplier would then be used to obtain 405-megacycle energy, and then a klystron frequency multiplier in which the 405-megacycle energy is multiplied to 3,645-megacycles. The 5- and 45-megacycle energies would then be inductively coupled to the base leads of the klystron amplifier, in the place occupied by the two Hartley oscillators in Fig. 1. The 3,645-megacycle energy would be the carrier, and would be introduced into the input cavity of the klystron amplifier instead of the reflex klystron output, as in the configuration of Fig. 1.

To circumvent the inconvenience of retuning the output cavity of the klystron for each separate sideband, it is proposed that either a special tube be used which would work into a waveguide, or that a conventional tube be used, and that the output cavity be heavily loaded to decrease its Q, and consequently increase the bandwidth of the output. With this wideband frequency spectrum of relatively high power, the unknown frequency might be compared with the standard frequency by means of the heterodyne oscillator principle [1]

In the first part of this paper, it will be shown by theoretical considerations that operation of a two-resonator klystron amplifier in this way will produce a broad band of microwave energy at the klystron output cavity. In the second part of the paper, the experimental verification will be presented.

THEORETICAL DEVELOPMENT

Beam Current at the Input Gap[2]

The beam accelerating voltage of the klystron amplifier will be the sum of the direct beam accelerating voltage and the two high-frequency alternating voltages which are superimposed on the dc accelerating voltage (see Fig. 1), or

$$E = E_0 + E_1 \sin \omega_1\tau + E_2 \sin \omega_2\tau, \qquad (1)$$

where

E_0 = direct-beam accelerating voltage,
$E_1 \sin \omega_1\tau$ = low-frequency alternating-beam voltage variation,
$E_2 \sin \omega_2\tau$ = high-frequency alternating-beam voltage variation,
τ = time.

Since only about six electrical degrees of $\omega_2\tau$ are required for electron transit from the gun to the input gap in a practical klystron design, beam bunching will

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[1] Massachusetts Institute of Technology Staff, "Applied Electronics," John Wiley and Sons, Inc., New York, N. Y., p. 697; 1943.

[2] The analysis presented in this and the succeeding section follows a procedure developed in detail by A. E. Harrison, "Klystron Tubes," McGraw-Hill Book Co., Inc., New York, N. Y., p. 23; 1947.

be slight compared to Child's law effects, so only the Child's law effects will be considered. At the instant before entering the input gap, the beam current $i(\tau)$ will be, from Child's law,

$$i(\tau) = K(E_0 + E_1 \sin \omega_1\tau + E_2 \sin \omega_2\tau)^{3/2},$$

where
K = proportionality constant, or perveance.

Dividing and multiplying by $E_0^{3/2}$, and defining $K_1 = KE_0^{3/2}$,

$$i(\tau) = K_1\left(1 + \frac{E_1}{E_0} \sin \omega_1\tau + \frac{E_2}{E_0} \sin \omega_2\tau\right)^{3/2}.$$

Expanding, and taking the first approximation, the expression for the input gap beam current is obtained:

$$i(\tau) = K_1(1 + i_1 \sin \omega_1\tau + i_2 \sin \omega_2\tau), \tag{2}$$

where

$$i_1 = \frac{3}{2}\frac{E_1}{E_0}$$

$$i_2 = \frac{3}{2}\frac{E_2}{E_0}.$$

Transit Time Between the Input and Output Gaps

Assuming zero input gap width, the velocity v of the beam electrons in the space between the input and output gaps will conform to the equation:

$$v = 5.94 \times 10^7 \sqrt{E_0 + E_1 \sin \omega_1\tau + E_2 \sin \omega_2\tau + E_3 \sin \omega_3\tau} \text{ cm/sec}$$

where

$$E_3 \sin \omega_3\tau = \text{microwave carrier voltage.}$$

The drift space is field free; therefore, no accelerating forces will be applied to the electrons in this region. The transit time T may be expressed:

$$T = \frac{d = \text{drift distance (cm)}}{v} \text{ sec.}$$

Expanding by the binomial theorem and taking the first approximation,

$$T = T_0 - T_1 \sin \omega_1\tau - T_2 \sin \omega_2\tau - T_3 \sin \omega_3\tau, \tag{3}$$

where

$$T_0 = \frac{d}{5.94 \times 10^7 E_0},$$

$$T_1 = \frac{E_1T_0}{2E_0}, \qquad T_2 = \frac{E_2T_0}{2E_0}, \qquad T_3 = \frac{E_3T_0}{2E_0}.$$

Beam Current at the Output Gap

The relationship between beam current at the input gap, transit time, and beam current at the output gap is illustrated in Fig. 2. From Fig. 2,

$$T_0 + t = \tau + T. \tag{4}$$

From the law of continuity of charge and assuming no beam debunching,

$$i(t) = \frac{i(\tau)}{\left(\frac{dt}{d\tau}\right)}, \tag{5}$$

where

$i(t)$ = beam current at the output gap,

$i(\tau)$ = beam current at the input gap.

Differentiating (4) and substituting in (5),

$$i(t) = \frac{i(\tau)}{1 + dT/d\tau}, \tag{6}$$

which is an equation expressed in terms of time at the output gap only.

The problem studied in this paper required an analysis of frequency sidebands at the output gap, and not the output gap beam current as a function of time, which is the form of (6). It is therefore profitable to transform (6) by means of the Laplace transform[3] into an expression that is a function of frequency.

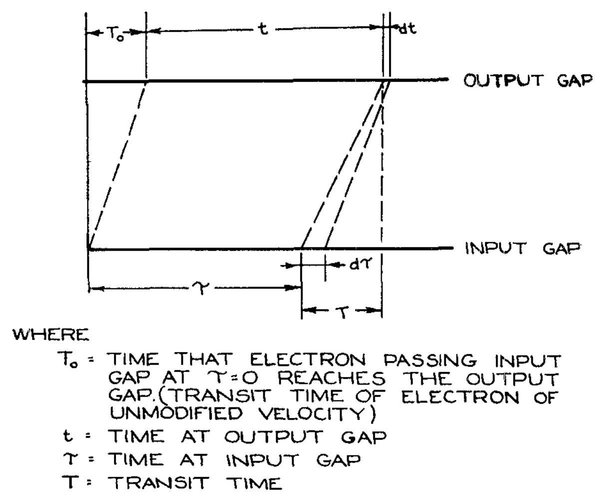

Fig. 2—Modified Applegate diagram.

Frequency Spectrum of Beam Current at Output Gap

By definition of the Laplace transform,

$$f(s) = \int_0^\infty F(t)e^{-st}dt, \tag{7}$$

or, for the problem at hand,

$$f(s) = \int_0^\infty \frac{i(\tau)}{1 + dT/d\tau} e^{-st}dt.$$

Making used of (4) and (5), (7) becomes

[3] H. Chestnut and R. W. Mayer, "Servomechanisms and Regulating System Design," John Wiley and Sons, New York, N. Y., vol. I, p. 110; 1951.

[4] G. N. Watson, "A Treatise on the Theory of Bessel Functions," The Macmillan Co., New York, N. Y., p. 22; 1948.

TABLE I

CONTRIBUTIONS TO SECOND UPPER SIDEBAND (3117.5 MC/SEC) WHEN $f_1=2.5$ MC/SEC, $f_2=12.5$ MC/SEC, $f_3=3112.5$ MC/SEC

n	$n\omega_1$ mc/sec	m	$m\omega_2$ mc/sec	p	$p\omega_3$ mc/sec	$n\omega_1+m\omega_2+p\omega_3$
2	$2\pi\times 5$	0	0	1	$2\pi\times3112.5$	$2\pi\times3117.5$
−3	$-2\pi\times 7.5$	1	$2\pi\times12.5$	1	$2\pi\times3112.5$	$2\pi\times3117.5$
7	$2\pi\times17.5$	−1	$2\pi\times12.5$	1	$2\pi\times3112.5$	$2\pi\times3117.5$

$$f(s) = \int_0^\infty i(\tau)e^{-s(\tau+T-T_0)}d\tau. \tag{8}$$

Substituting (2) and (3) in (8), and making use of Euler's formulas,

$$f(s) = K_1\int_0^\infty \left\{\left[1 + i_1\left(\frac{e^{j\omega_1\tau} - e^{-j\omega_1\tau}}{2j}\right) + i_2\left(\frac{e^{j\omega_2\tau} - e^{-j\omega_2\tau}}{2j}\right)\right] \exp\,(sT_1 \sin \omega_1\tau + sT_2 \sin \omega_2\tau + sT_3 \sin \omega_3\tau - s\tau)\right\} d\tau. \tag{9}$$

Making use of the Bessel identity,[4]

$$e^{jp \sin \omega\tau} = \sum_{n=-\infty}^{\infty} J_n(p)e^{jn\omega\tau},$$

and expanding, (9) becomes

As an example, Table I, above, lists some terms that contribute to the magnitude of the second upper sideband $(\omega_3+2\omega_1)$, when ω_1, ω_2, and ω_3 have the values 2.5, 12.5, and 3,112.5 megacycles per second, respectively. These frequencies are approximately the values used in the experimental work for this paper.

We now make the definitions:

$$q\omega_1 = (n\omega_1 + m\omega_2 + p\omega_3) \tag{12}$$

and

$$K_q(s) = \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} \cdot J_n\left(\frac{s}{j}\,T_1\right)J_m\left(\frac{s}{j}\,T_2\right)J_p\left(\frac{s}{j}\,T_3\right). \tag{13}$$

The relationship between q, n, m, and p in (13) must be as defined in (12), and q must be an integer.

Since $K_q(s)$ is independent of τ, the integration indicated in (11) may be performed to give

$$\begin{aligned} f(s) = K_1\int_0^\infty &\left\{\left[\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} J_n\left(\frac{s}{j}\,T_1\right)J_m\left(\frac{s}{j}\,T_2\right)J_p\left(\frac{s}{j}\,T_3\right)e^{j\tau(n\omega_1+m\omega_2+p\omega_3)}\right]\right.\\ &+\frac{i_1}{2j}\left[\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} J_n\left(\frac{s}{j}\,T_1\right)J_m\left(\frac{s}{j}\,T_2\right)J_p\left(\frac{s}{j}\,T_3\right)e^{j\tau[(n+1)\omega_1+m\omega_2+p\omega_3]}\right]\\ &-\frac{i_1}{2j}\left[\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} J_n\left(\frac{s}{j}\,T_1\right)J_m\left(\frac{s}{j}\,T_2\right)J_p\left(\frac{s}{j}\,T_3\right)e^{j\tau[(n-1)\omega_1+m\omega_2+p\omega_3]}\right]\\ &+\frac{i_2}{2j}\left[\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} J_n\left(\frac{s}{j}\,T_1\right)J_m\left(\frac{s}{j}\,T_2\right)J_p\left(\frac{s}{j}\,T_3\right)e^{j\tau[n\omega_1+(m+1)\omega_2+p\omega_3]}\right]\\ &\left.-\frac{i_2}{2j}\left[\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} J_n\left(\frac{s}{j}\,T_1\right)J_m\left(\frac{s}{j}\,T_2\right)J_p\left(\frac{s}{j}\,T_3\right)e^{j\tau[n\omega_1+(m-1)\omega_2+p\omega_3]}\right]\right\} e^{-s\tau}d\tau. \end{aligned} \tag{10}$$

Consider the first of the five terms in the integrand of (10). Call this term $f_1(s)$.

$$f_1(s) = \int_0^\infty \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} J_n\left(\frac{s}{j}\,T_1\right)J_m\left(\frac{s}{j}\,T_2\right)J_p\left(\frac{s}{j}\,T_3\right)e^{j\tau(n\omega_1+m\omega_2+p\omega_3)}e^{-s\tau}d\tau. \tag{11}$$

Eq. (11) is made up of two terms: a magnitude portion consisting of the Bessel coefficients, and a periodic portion consisting of the term $e^{j(n\omega_1+m\omega_2+p\omega_3)}$. The angular velocity of this function will be $(n\omega_1+m\omega_2+p\omega_3)$. Since n, m, and p each take on all integer values between $-\infty$ and ∞, there will be an infinite number of terms contributing to each angular velocity.

$$f_1(s) = \cdots + \frac{K_{q-b}(s)}{s - jq_{-b}\omega_1} + \cdots + K_0(s) + \cdots + \frac{K_{qb}(s)}{s + jq_b\omega_1} + \cdots +, \tag{14}$$

where the summation is over b, and b takes on all values from $-\infty$ to ∞.

If the right-hand side of (14) is multiplied by $(s+jq_b\omega_1)$, and s is then set equal to $-jq_b\omega_1$, all terms in the right-hand side of (14) will drop out except $K_{qb}(s)$. The real part of this term, $K_{qb}(s)$, will then represent the magnitude of the q_b sideband.

We now rewrite (10) with (12) and (13) substituted therein:

$$f(s) = K_1 \int_0^\infty \Bigg[\sum_{q=-\infty}^{\infty} K_q(s) e^{jq\omega_1\tau} + \frac{i_1}{2j} \sum_{q=-\infty}^{\infty} K_q(s) e^{j\omega_1\tau(q+1)} - \frac{i_1}{2j} \sum_{q=-\infty}^{\infty} K_q(s) e^{j\omega_1\tau(q-1)} + \frac{i_2}{2j} \sum_{q=-\infty}^{\infty} K_q(s) e^{j\tau(q\omega_1+\omega_2)} - \frac{i_2}{2j} \sum_{q=-\infty}^{\infty} K_q(s) e^{j\tau(q\omega_1-\omega_2)} \Bigg] e^{-s\tau} d\tau. \quad (15)$$

Taking advantage of the complex conjugacy of $K_{-n}(s)$ and $K_n(s)$, and using the familiar sine-exponential identity, (15) may be rewritten to give:

$$f(s) = K_1 \int_0^\infty \Bigg[\sum_{q=0}^{\infty} K_q(s) e^{jq\omega_1\tau} + i_1 \sum_{q=0}^{\infty} K_q(s) \sin \omega_1(q+1)\tau + i_1 \sum_{q=0}^{\infty} K_q(s) \sin \omega_1(q-1)\tau + i_2 \sum_{q=0}^{\infty} K_q(s) \sin (q\omega_1+\omega_2)\tau + i_2 \sum_{q=0}^{\infty} K_q(s) \sin (q\omega_1-\omega_2)\tau \Bigg] e^{-s\tau} d\tau. \quad (16)$$

In (16), the second, third, fourth, and fifth terms are the familiar amplitude-modulation sidebands.

In the problem studied in the experimental work of this paper, $\omega_2 = 5\omega_1$. Eq. (16) may be written entirely in terms of ω_1 as follows:

$$f(s) = K_1 \int_0^\infty \Bigg[\sum_{q=0}^{\infty} K_q(s) e^{jq\omega_1\tau} + i_1 \sum_{q=0}^{\infty} K_{q-1}(s) \sin \omega_1 q\tau + i_1 \sum_{q=0}^{\infty} K_{q+1}(s) \sin \omega_1 q\tau + i_2 \sum_{q=0}^{\infty} K_{q-5}(s) \sin \omega_1 q\tau - i_2 \sum_{q=0}^{\infty} K_{q+5}(s) \sin \omega_1 q\tau \Bigg] e^{-s\tau} d\tau. \quad (17)$$

Determination of the Coefficients

In order that a frequency spectrum be calculated from (12), (13), and (17), the values of the coefficients, T_0, T_1, T_2, T_3, i_1, and i_2 must be determined. These coefficients are all functions of E_1, E_2, and E_3, as defined by (3), so, in order to calculate the coefficients, these three voltages must first be known.

The values of E_0, E_1, and E_2 as used in the experimental work were approximately:

$$E_0 = 660 \text{ volts}, \quad E_1 = 34 \text{ volts}, \quad E_2 = 44 \text{ volts}. \quad (18)$$

The value of E_3, the input gap voltage of the klystron amplifier, could not be measured directly in the laboratory. This makes necessary the examination of (10) to determine the way in which the magnitude of E_3 affects the magnitudes of the sidebands. Eq. (10) shows that the only calculation which contains E_3 is the argument of the J_p Bessel coefficient. Since E_3 is a constant and $p=1$ for all work of this paper, J_p is also a constant. Since J_p multiplies each sideband magnitude equally, the conclusion is reached that the magnitude of E_3 does not affect the ratio of magnitudes of the sidebands. The value of E_3 does, of course, affect the relative magnitudes of the sidebands with respect to the dc beam voltage and, consequently, the absolute value of the various sidebands.

Since we are interested in the relative magnitudes of the sidebands, the value of E_3 may be properly left undetermined.

From (2), (3), and (18),

$$i_1 = .0773, \quad T_0 = 1.62 \times 10^{-9}, \quad T_1 = 4.18 \times 10^{-11}, \quad i_2 = .1, \quad T_2 = 5.4 \times 10^{-11}. \quad (19)$$

Since the value of E_3 is to be left undetermined, we may, without loss of generality, set the J_p term of (13) equal to 1.

Determination of Significant Terms

In order to obtain quantitative results from (17), it is necessary to determine how many of the terms of the infinite series must be included in the calculations. The number of terms required will depend in turn on the required accuracy. We will define the required accuracy as one per cent of the greatest sideband magnitude. We shall interpret this to mean that all terms of (19) may be ignored which contribute less than one per cent of the magnitude of the greatest sideband to the particular sideband with which it is associated.

Substituting $s=j\omega$, $\omega = 2\pi \times 3{,}112.5 \times 10^6$, and the values of T_1 and T_2 from (16), we find that

$$J_n(\omega T_1) = J_n(.815), \qquad J_m(\omega T_2) = J_m(1.052).$$

Since the greatest single contribution to the fundamental frequency magnitude is made by the product in which both n and m equal zero, all terms in which either $J_n(.815)$ or $J_m(1.052)$ reach values less than one per cent of their value for J_0 will render that term insignificant. Calculations show that for $-3>n$ or $m<3$, then $J_n(.815)<.01J_0(.815)$ and $J_m(1.052)<.01J_0(1.052)$. Therefore (17) will be considered to have the limits of summation -3 to $+3$.

Sample Calculation

As an example, the contributions of these five terms to the seventh upper sideband ($\omega_3+7\omega_1$) will be made. For reasons already explained, only values of n and m in the ranges $-3<n<3$, and $-3<m<3$ will be used. Rewriting (12) and (13) for reference, with $s=j\omega$ and the limits extending from 3 to -3,

$$q\omega_1 = (n\omega_1 + m\omega_2 + p\omega_3), \qquad (12)$$

$$K_q(j\omega) = \sum_{n=-3}^{\infty} \sum_{m=-3}^{\infty} J_n(\omega T_1)J_m(\omega T_2)J_1(\omega T_3). \qquad (13a)$$

In the first term the values of n and m will be

$$q\omega_1 = (\omega_3 + 7\omega_1) = (n\omega_1 + m\omega_2 + p\omega_3),$$

and, since $\omega_2=5\omega_1$, the only values of n and m in the required ranges which will satisfy this equality are

$$(n = 2, m = 1) \quad \text{and} \quad (n = -3, m = 2).$$

In the second term of (17), the values of n and m will be determined by the equation

$$\omega_1(q - 1) = (\omega_3 + 7\omega_1) - \omega_1 = (n\omega_1 + m\omega_2 + p\omega_3);$$

or, since $p=1$,

$$6\omega_1 = n\omega_1 + m\omega_2.$$

This will give the value

$$n = 1, \qquad m = 1.$$

The values of n and m to give contributions from the last three products may be similarly calculated. Table II lists the values of the terms contributing to the seventh upper sideband, with ωT_1 and ωT_2 equal to (.815) and (1.052) respectively. The cosine and sine coefficients calculated in Table II will add at right angles to give

$$f_7 = \sqrt{(.0344)^2 + (.02155)^2} = .0406.$$

If we had taken only the first term instead of all terms, the above calculation shows that an error of about 17 per cent would have been introduced in the seventh upper sideband magnitude. It must be remembered that we are interested in graphical accuracy, however, so the actual allowable error would be about one per cent of the value of the greatest magnitude plotted. In this study the magnitude of the fundamental is the greatest, or approximately

$$J_0(.815)J_0(1.052) = (.8407)(.7419) = .624.$$

The ratio of the seventh sideband error to the fundamental magnitude would be

$$E = \frac{.0406 - .0344}{.624} = .01.$$

In the spectrum considered, the last four terms of (20) gave their greatest relative contribution to the seventh upper sideband. Since the contribution of these last four terms is smaller than the maximum allowable error, these four terms will be ignored in the calculation of the frequency spectrum, and the equation which will be used to calculate the frequency spectrum will thus be, from (17), (12), and (13a),

$$f(s) = K_1 \int_0^{\infty} \sum_{n=-3}^{3} \sum_{m=-3}^{3} J_n(.815)J_m(1.052) e^{j\tau(n\omega_1+m\omega_2+\omega_3)}d\tau. \qquad (20)$$

The conclusion that the last four terms in (19) may be ignored is equivalent to neglecting the amplitude variations of the beam current. In other words, it has been shown that the variation of the beam current in accordance with Child's law is unimportant for the magnitude of modulation chosen for this example.

The frequency spectrum as calculated from (20) is plotted in Fig. 3 on the facing page.

Experimental Study of the Problem

The circuitry used to perform the experimental work is shown in Fig. 1. In the method used, the leads to the base of a klystron amplifier were inductively coupled to two Hartley oscillators, tuned to frequencies of 2.5 and

TABLE II

Term	(1) n	(2) $J_n(.815)$	(3) m	(4) $J_m(1.052)$	(2)×(4)	(i_1)	(i_2)	Contribution
1	2 −3	.0785 −.0108	1 2	.4568 .126	.0358 −.00136			.0358 −.00136
2	1	.3746	1	.4565	.171	.077		.01316
3	3 −2	.0108 .0785	1 2	.4565 .126	.00492 .00989	.077 .077		.00038 .00076
4	2 −3	.0785 −.0108	0 1	.7419 .4565	.0582 .00493		.109 .109	.00634 −.00054
5	2 −3	.0785 −.0108	2 3	.126 .0226	.00989 .00024		.109 .109	.00108 −.00003
					Total Cosine Coefficient			.0344
					Total Sine Coefficient			.02155

12.5 mc, respectively. This coupling to the Hartley oscillators formed the high-frequency variations of the direct beam voltage. The microwave energy was supplied to the input cavity in the usual manner.

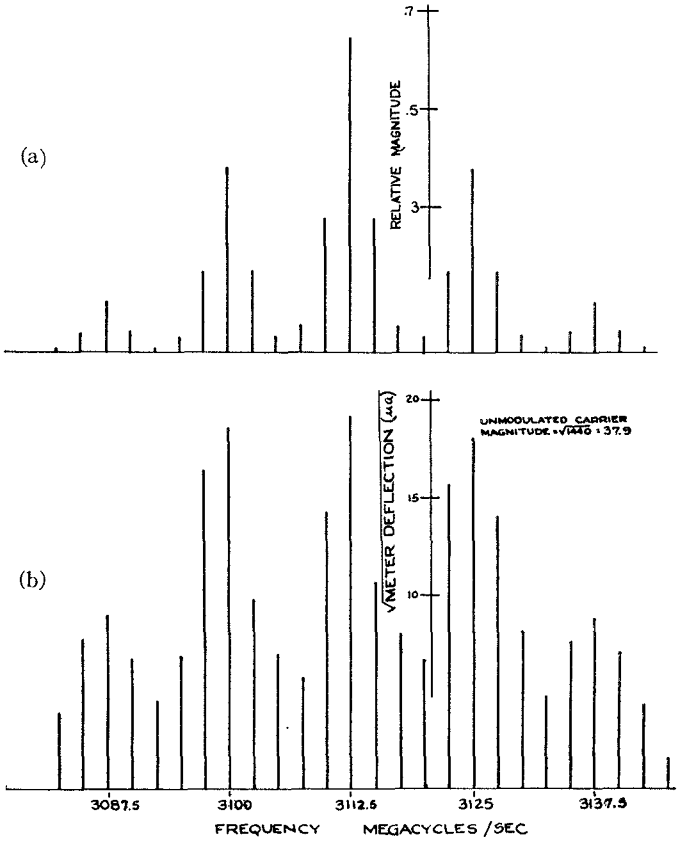

Fig. 3—Frequency spectra: (a) calculated, (b) experimental.

A tabulation of results is presented in Fig. 3. The plotted amplitudes are proportional to the square root of the meter deflections because the crystal detector is a square law device and the meter readings are proportional to the square of the sideband magnitudes.[5]

The data plotted in Fig. 4 were obtained during some early tests of the double-modulation method, and illustrate the wide spectrum that can be obtained with modulation voltages approximately two times the values used for the experimental data in Fig. 3. The analysis did not show good correspondence with experimental data for these large modulating voltages. Note that a different carrier frequency was used in the earlier tests, and the higher modulation frequency was six times the lower frequency instead of five times the lower frequency, as was used in the later work.

Comparison of Calculated and Experimental Data

Referring again to Fig. 3, it will be seen that the correlation between theoretical and experimentally determined spectra was good. Both spectra clearly show the wide distribution of evenly spaced sidebands, as would be required for a satisfactory frequency standard. The most noticeable difference between the calculated and experimental spectra is the greater spread of energy over the entire spectrum in the case of the experimental sidebands. This was undoubtedly partly due to the very great amount of harmonics present in the output of the Hartley oscillators, but it was also due to the great higher order effects of the klystron which make it so rich in harmonics.

[5] C. G. Montgomery, "Techniques of Microwave Measurement," McGraw-Hill Book Co., New York, N. Y., p. 496; 1947.

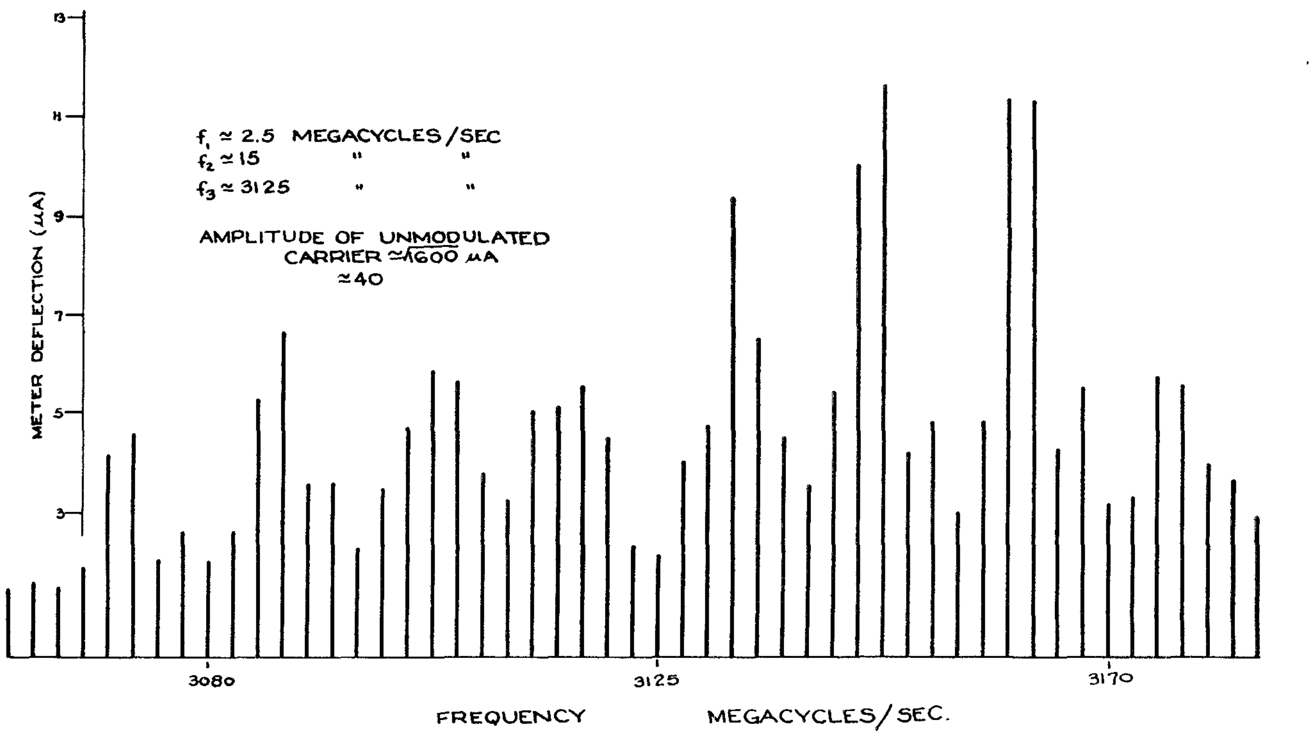

Fig. 4—Frequency spectrum with large modulating voltages.

The magnitudes of the sideband components would be much smaller when the output cavity is loaded down in order to increase the bandwidth than they were in the experimental work which was performed for this paper. In an actual case, a bandwidth of about a hundred megacycles would probably be desired, which would require loading the output cavity until its equivalent Q is about sixty,[6] as compared with the Q of about a thousand in the cavity configuration used for this paper. This would mean that the magnitudes of the various sidebands would be approximately 7 to 10 per cent of the values obtained in the experimental work of this paper. The tube used in this work was a very poor one; a large share of this decrease in magnitude could be regained by using a good klystron and higher voltages.

[6] Massachusetts Institute of Technology Staff, *op. cit.*, p. 509.

Conclusion

It is believed that this preliminary investigation has shown that this method will produce a frequency spectrum that is admirably suited for use for a microwave frequency standard. It has been shown that this scheme will produce a wide band of microwave voltages of reasonable magnitude and accurately determinable frequencies. With a single carrier frequency, an operating range of ten or fifteen times the lowest modulating frequency may be easily obtained on each side of the carrier frequency.

A Turnstile Polarizer for Rain Cancellation

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Summary—**This paper describes a rain canceller using a turnstile junction to provide the necessary polarization of the fields. It discusses the system from the point of view of adapting an existing radar feed system to one that will permit the reduction of rain return. The turnstile junction is described, and it is shown to have properties that meet the requirements for obtaining circular or elliptical polarizations. Along with this description, the theory behind the effect of elliptical and circular polarization on rain return is discussed in detail as well as the practical limits such a theory can be put to. Other considerations such as aspect ratio of antennas and screening for the antenna surfaces are touched upon.**

THE DELETERIOUS effect of rain on radar systems has long been a troublesome condition. This effect is well known and has been the object of much study. The energy lost to a target is lost in two ways:

1. The rain absorbs the rf and converts it to heat,
2. The spherical drops reflect the energy and cause it to appear as scintillating high-level noise.

The first condition is a function of wavelength:

$W \alpha S f(a, \lambda)$ = energy lost,

S = pointing vector,

a = radius of a spherical drop,

λ = wavelength in air.

The loss is directly proportional to frequency. At 10 cm and above, the attenuation is negligible, and even down to 3 cm it is not too serious; but for $\lambda < 3$ cm the attenuation goes from 0.01 db/km for a drizzle to 1 db/km for excessive rain. For our case, this attenuation is unavoidable and must be accepted as reduction in system performance. However, the second case of rain scattering the incident signal can be remedied to some extent. This "rain clutter" can be extremely troublesome for targets of small cross section when the beam is pencil or fan shaped.

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There is one saving feature to this entire problem, and that is the unique property of the rain target itself, namely, its shape. Since rain drops tend to be spherical, the intensity and phase of the reflection from the target do not depend on the direction in which the incident beam is polarized. When these spheres are struck by a circularly polarized plane wave, the scattered wave which is returned is also circularly polarized and just the sense of rotation of the vector is changed (see Fig. 1).

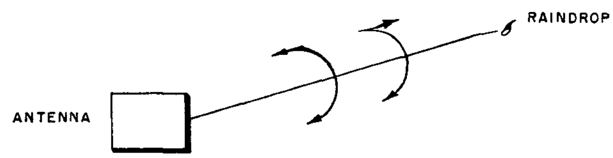

Fig. 1—Principle of rain cancellation.

The sense is defined as the direction of rotation as seen by an observer watching an oncoming signal. Circular polarizers are so designed that this return, which is in the opposite sense from the impinging signal, will not be accepted by the feed system. A simple device which will manufacture circular polarization in one sense and